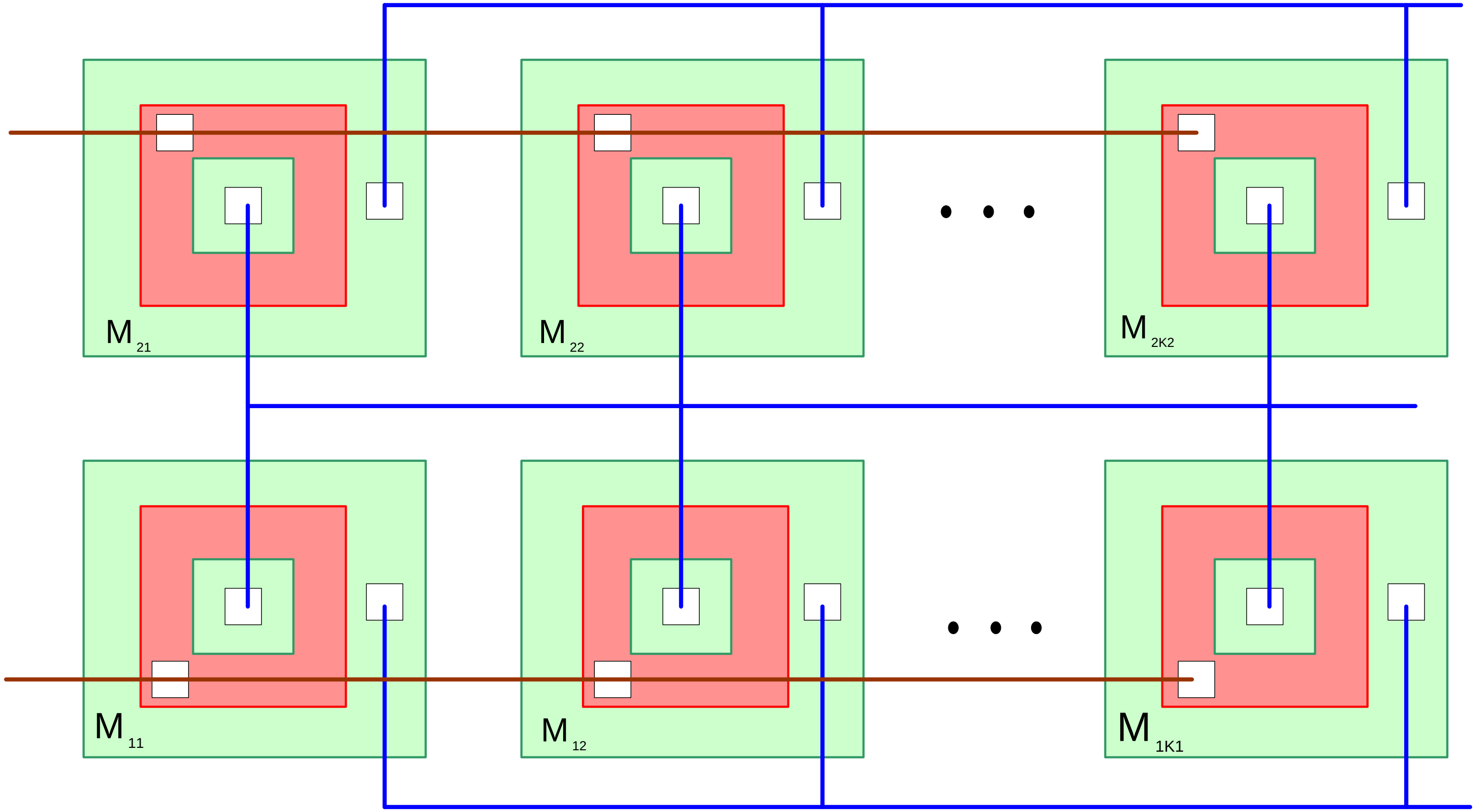
EE 508 Lecture 36

Noise in Filters
Dynamic Range

Review from last lecture

How high can I_0 be?

Consider concentric layouts for M_1 and M_2

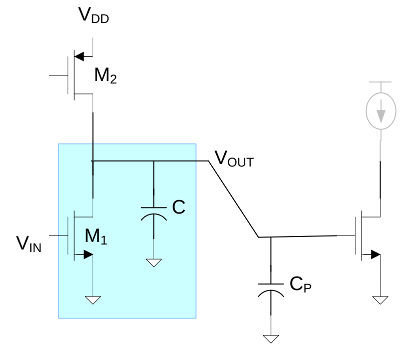


Individual segments can be a little bigger than minimum sized w/o major change in performance

May select K_1=K_2=1

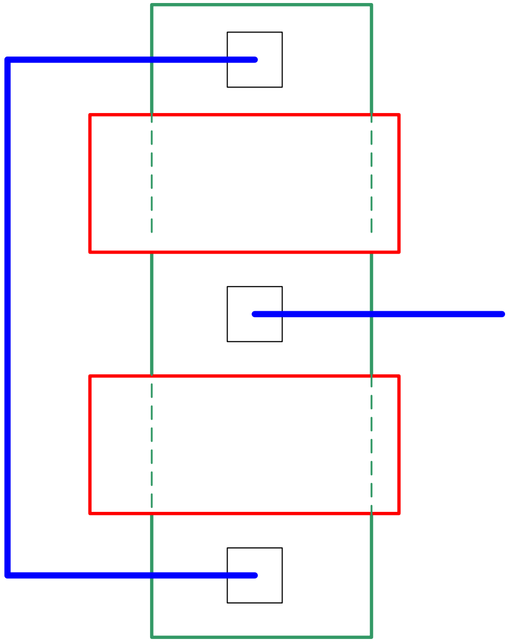
Review from last lecture

How high can I_0 be?



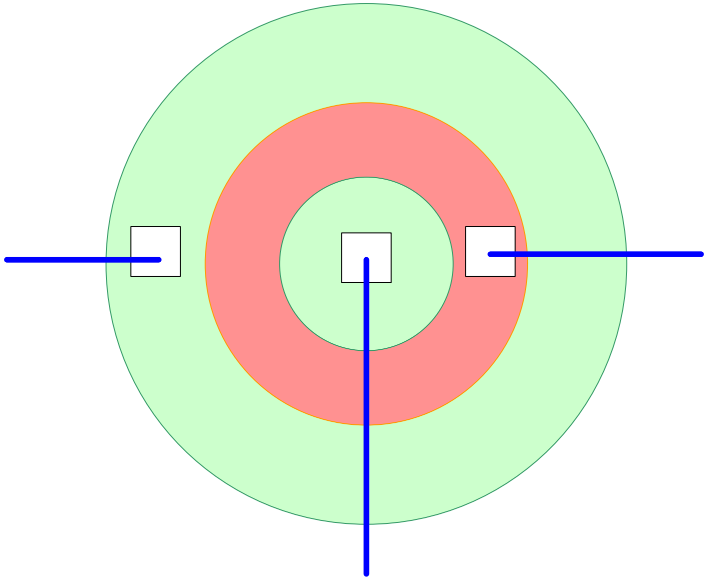
Other layouts for enhancing speed of operation

Goal: reduce area and perimeter on drain



Shared-drain structure

(but would not be applicable if one device in well and one outside of well)

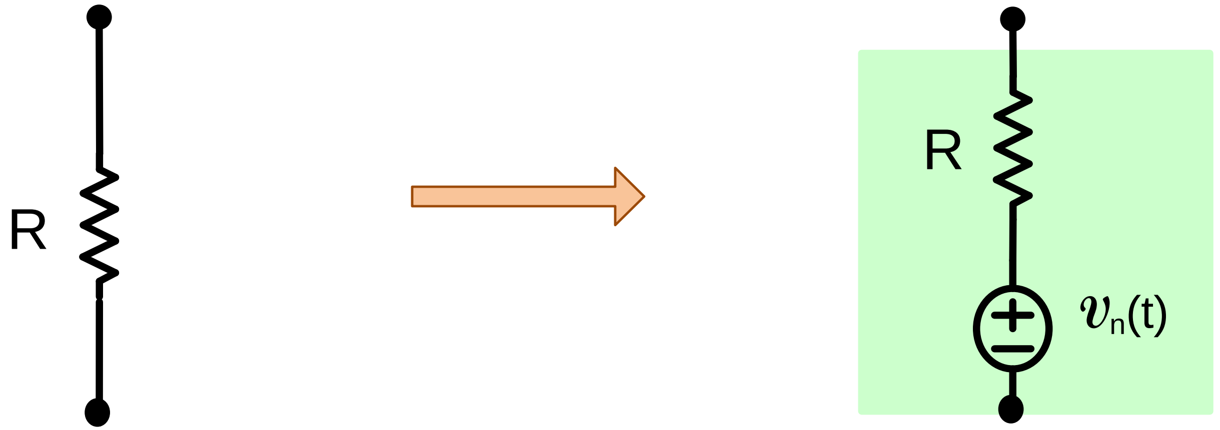


Circular-concentric structure

Though the reduced size drain structures work very well, CAD support may be limited for layout, simulation, and extraction

Noise is a random time-domain signal that characterizes movement of electrons in devices

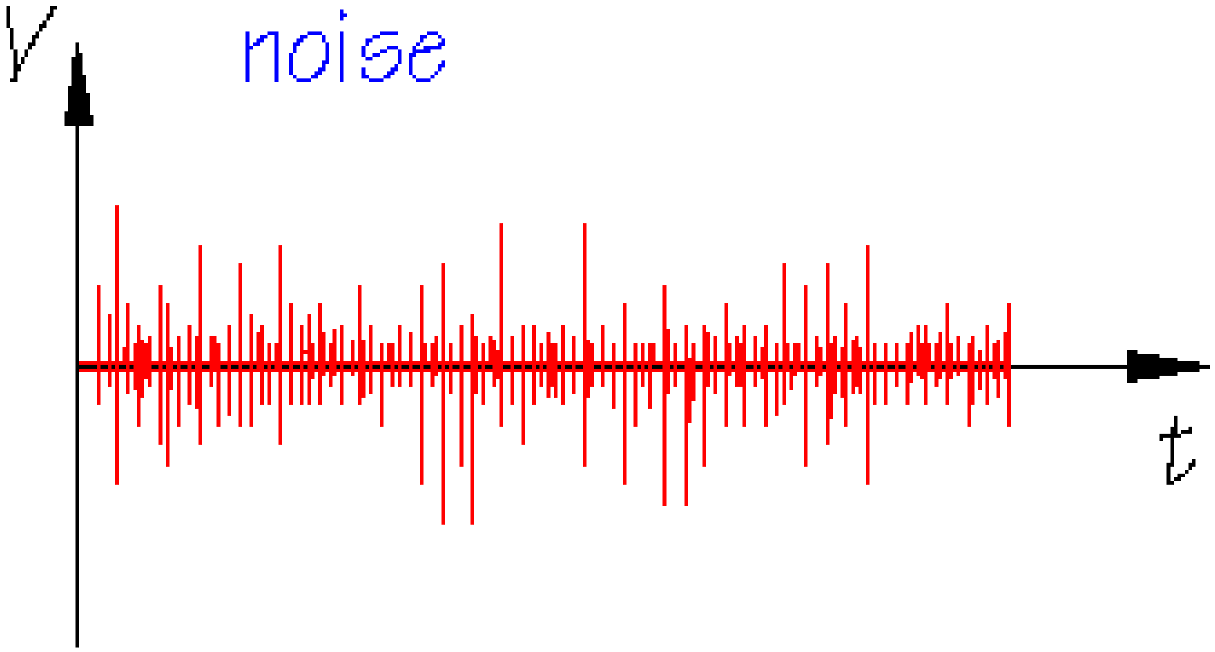
Example: Noise in Resistors



Ideal Resistor

Model of noise in a Resistor

Typical noise waveform for a resistor

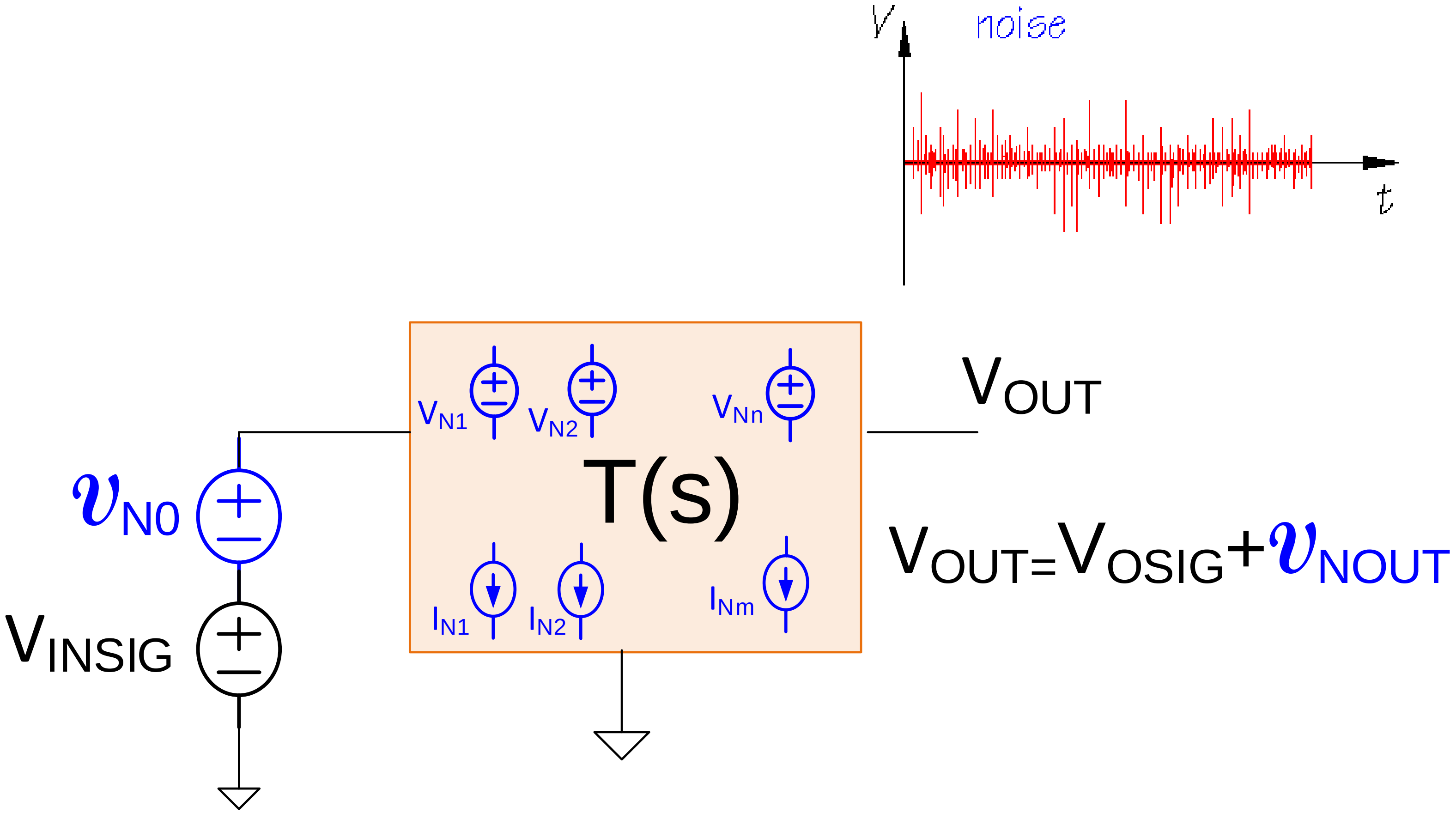


Noise sources in electronic devices are time-domain sources and can be modeled with independent voltage and current sources

Noise sources have a polarity though the statistical characteristics are independent of how the polarity is assigned

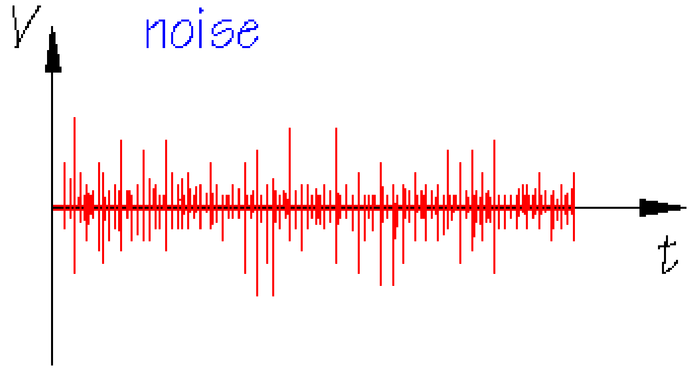
Noise is often quantified by the corresponding RMS value of the noise voltage or current at a node or branch in a circuit

Noise in a System



- Often many noises sources present
- One can be corrupting the input and others are internal to the system
- Noises sources often sufficiently small that superposition can be applied to determine the combined effects of all noise sources on v_{NOUT}

Characterization of a Noise Signal

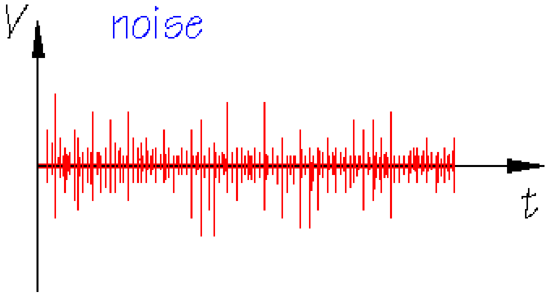


Noise naturally characterized by its RMS value

$$v_{RMS} = \lim_{T \to \infty} \int_{t_1}^{t_1+T} v^2(t)\,dt$$

Noise sources in electronic circuits

Resistors, Transistors, and Diodes all have one or more internal noise sources



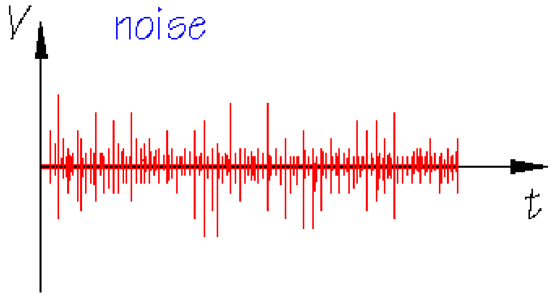
Capacitors and Inductors are noiseless

The presence of noise sources in devices is the only reason that input signals in filters are not made arbitrarily small to reduce effects of nonlinearity to arbitrarily small levels

The concept of “Dynamic Range” is used to characterize how small of input signals can be practically used in filters

To achieve acceptable linearity in a filter, the designer should provide just enough “dynamic range” to satisfy the requirements of an application. Any extra dynamic range will invariably come at the expense of increased design efforts, cost, complexity, and power dissipation

Dynamic Range

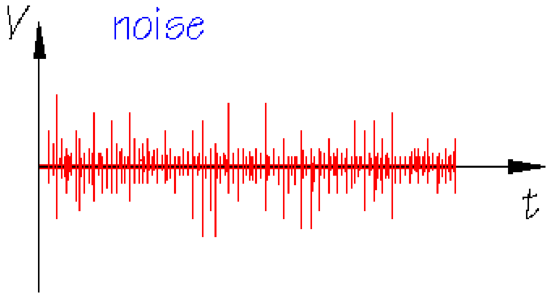


From Wikipedia:

“Dynamic range is the ratio of a specified maximum level of a parameter (e.g. quantity), such as power, current, voltage, or frequency, to the minimum detectable value of that parameter “

- The maximum level of such a quantity is strongly dependent upon the distortion acceptable in a particular application
- This value may be dependent upon frequency
- The minimum detectable value of a quantity may be dependent upon application
  Some authors interpret the minimum detectable value to be the RMS value of the quantity when the input signal is zero
- The use of a single value for the DR for a filter without knowing the specific applications is of questionable use

Dynamic Range



From Allen and Holberg:

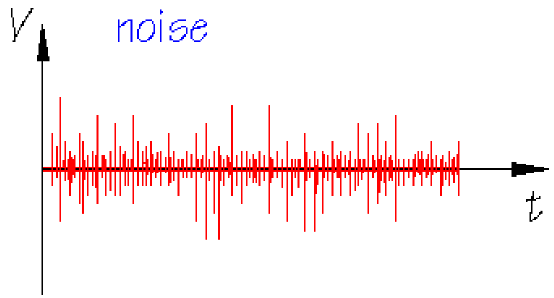
"whereas noise imposes a lower limit on the range of signal amplitudes that can be meaningfully processed by a circuit, linearity often imposes the upper limit. The difference between them is the dynamic range"

From Gregorian and Temes: (in the context of op amp circuits)

"Due to the limited linear range of the op-amp, there is a maximum input signal amplitude, $V_{in,max}$ which the device can handle without generating an excessive amount of nonlinear distortion. …… Due to spurious signals (noise, clock feedthrough, low-level distortion such as crossover distortion, etc.) there is also a minimum input signal $V_{in,min}$ which still does now drown in noise and distortion. The dynamic range of the op amp is then defined as $20\log_{10}\left(V_{in,\max}/V_{in,\min}\right)$ measured in decibels."

Numerous definitions for DR include some "qualitative" terms in the definition making it difficult to identify a universally accepted definition of the DR though the concept is useful

Dynamic Range

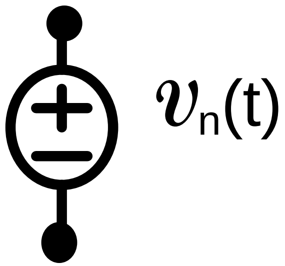


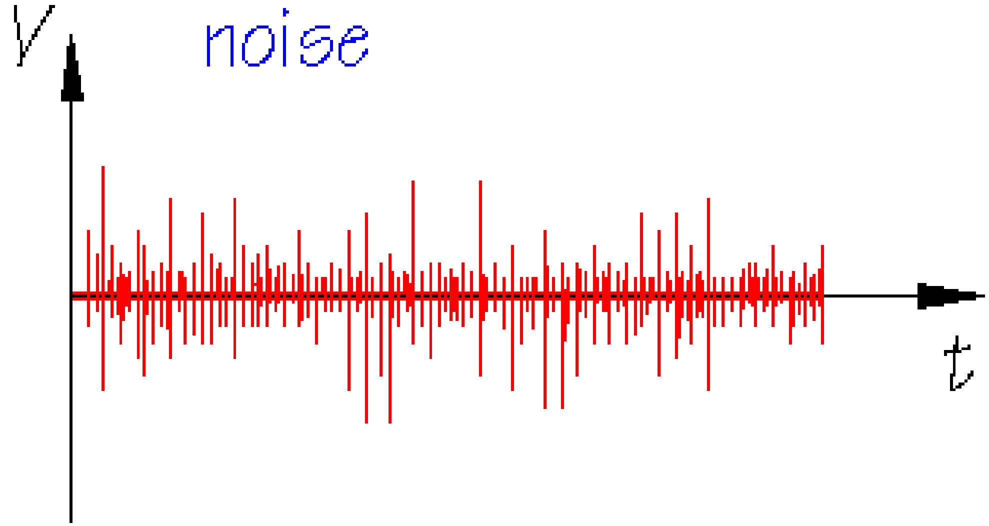
Numerous definitions for DR include some “qualitative” terms in the definition making it difficult to identify a universally accepted definition of the DR though the concept is useful

SNDR is a metric that is rigorously defined that captures some of the DR properties

Though the concept of DR is often not discussed rigorously and though there are various definitions of DR, Dynamic Range should be the primary driver of signal swing, power dissipation, and architecture selection not only in filter circuits but in analog circuit design in general

Statistical Characterization of Noise





If $v_n(t_1)$ is a sample of $v_n(t)$, then $v_n(t_1)$ is a random variable

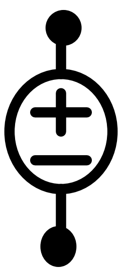
For almost all noise sources, the distribution of $v_n(t_1)$ is zero mean and often Gaussian

For many noise sources, if $v_n(t_1)$ and $v_n(t_2)$ are two distinct samples with $t_1 \neq t_2$, these random variables are identically distributed and uncorrelated (iid)

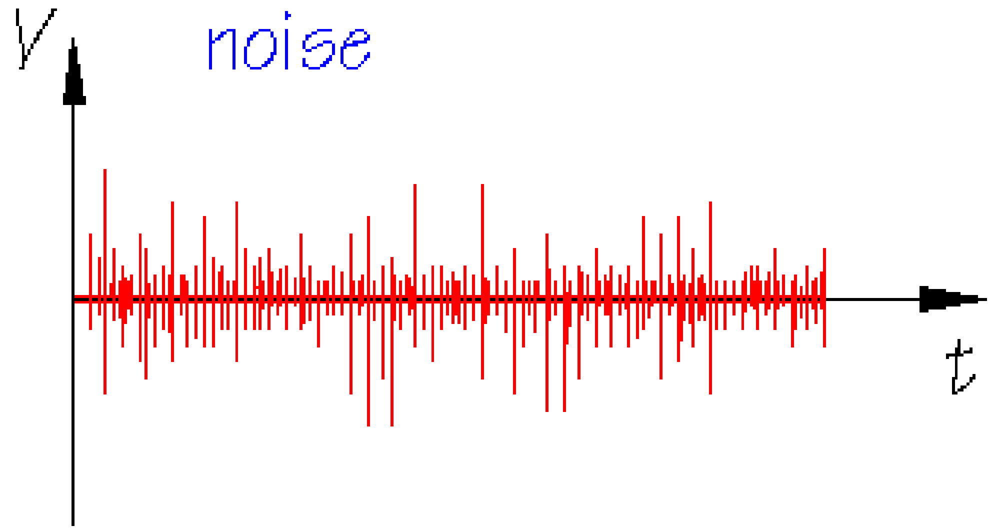
Noise (voltage) is also characterized by how it is distributed throughout the frequency spectrum by its power spectral density, S, or voltage spectral density S_V

Thus noise is characterized by both S and the amplitude distribution function

Statistical Characterization of Noise



$\mathcal{v}_n(t)$



The RMS noise voltage in the frequency band $[f_1,f_2]$ is given by the expression

$$\mathcal{v}_{RMS}\left(f_1,f_2\right)=\sqrt{\left(\int_{f_1}^{f_2}\mathcal{S}df\right)}$$

$$S = S_V^2$$

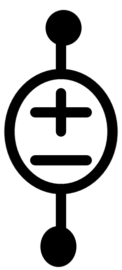
or

$$S = S_I^2$$

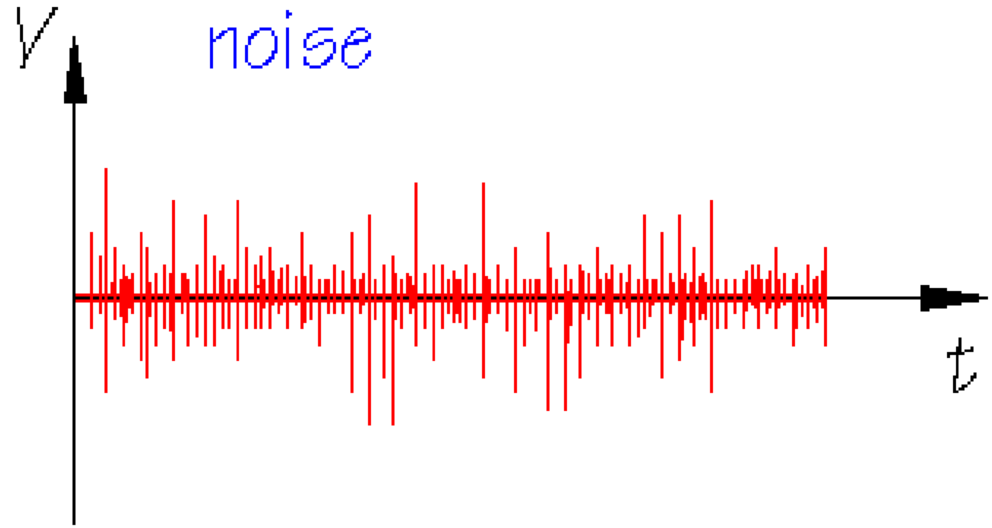
And the total RMS noise voltage is given by the expression

$$\mathcal{v}_{RMS}=\sqrt{\left(\int_{0}^{\infty}\mathcal{S}df\right)}$$

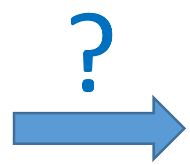
Statistical Characterization of Noise



$v_n(t)$



$$v_{RMS} = \sqrt{\left(\int_0^{\infty} S df\right)}$$

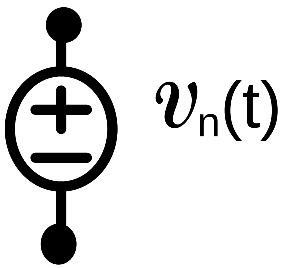


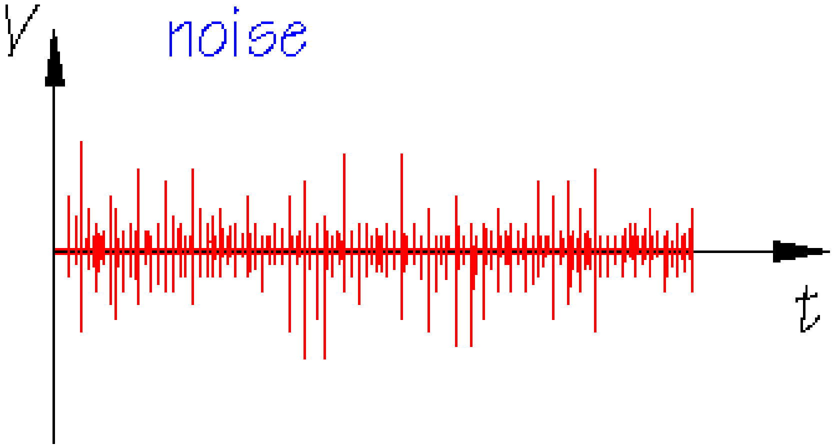
$$v_{RMS} = \lim_{T\to\infty} \int_{t_1}^{t_1+T} v^2(t)dt$$

Parseval's Theorem

$$\sqrt{\int_{f=0}^{\infty} Sdf} = \lim_{T\to\infty} \int_{t_1}^{t_1+T} v^2(t)dt$$

Statistical Characterization of Noise

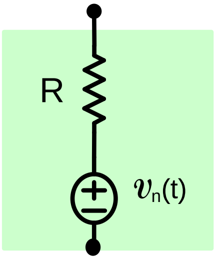




If the spectrum is flat, then the noise is termed “white” noise

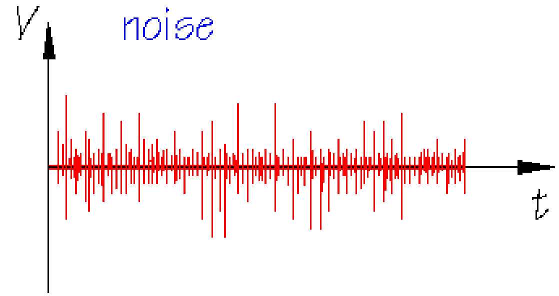
White noise can have an amplitude distribution that is Gaussian or non-Gaussian

For a resistor, the noise spectrum is white (over a very wide frequency range), the amplitude distribution is Gaussian, and any two distinct samples are iid.



$$S = 4kTR \quad (V^2 \ / \ Hz \ \ or \ \ V^2 \ \text{sec})$$

Dynamic Range



Often for audio filters, the DR is defined to be the ratio at the output between that due to a signal at 1% THD to the RMS noise voltage with the actual output spectrum multiplied by that of a C-Message bandpass filter

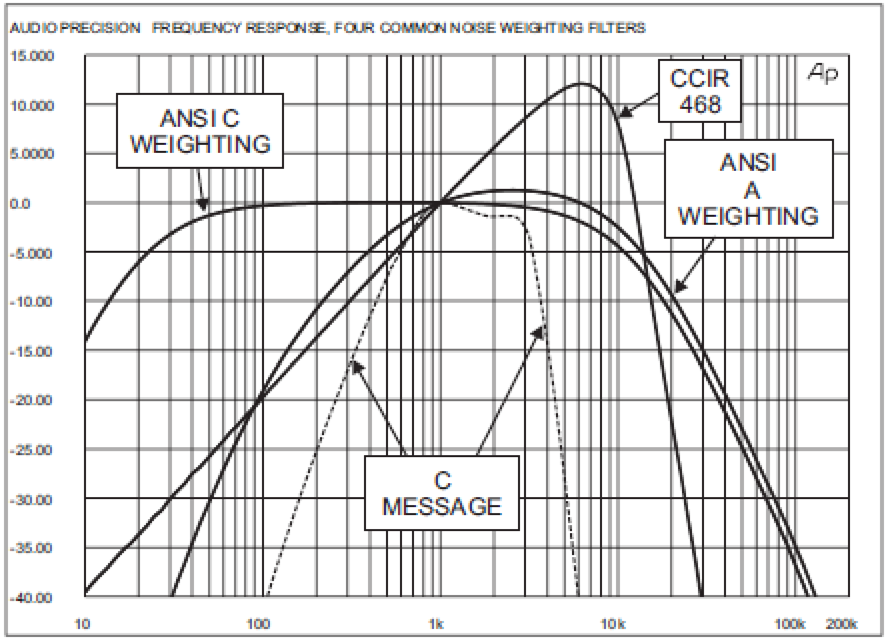
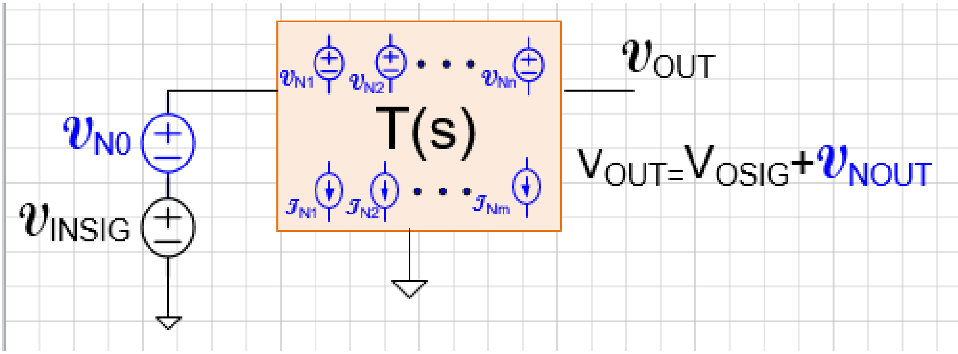


Figure 5. Weighting filter responses, actual measurements. Note that ANSI and C weighting filters are undefined above 20 kHz.

From "Audio Measurement Handbook" by Bob Metzlar

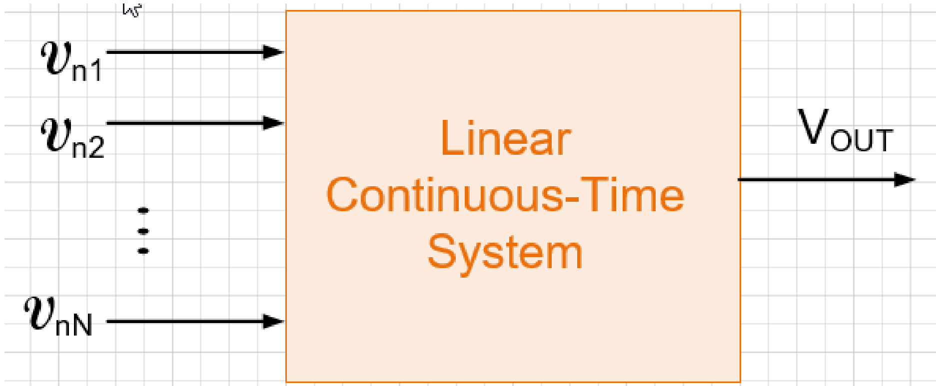
Analysis of Noise in Filter Circuits

Consider a filter circuit with N noise voltage sources (can be easily modified to include both noise voltage and current sources)



N=n+1

The noise sources can be represented by the block diagram shown below

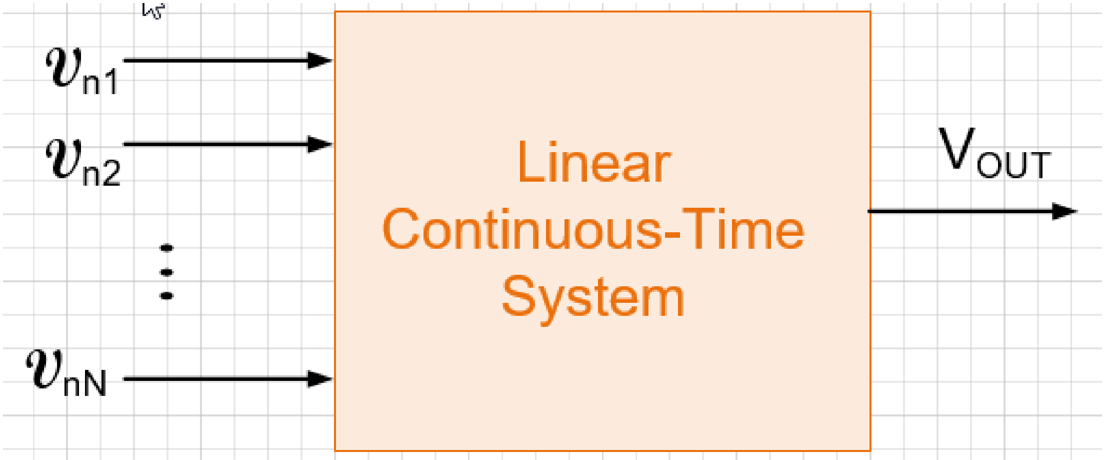


Assume $T_k(s)$ is the transfer function from the kth source to the output

By superposition

$$V_{OUT}(s) = \sum_{i=1}^{N} T_i(s) V_i(s)$$

Analysis of Noise in Filter Circuits



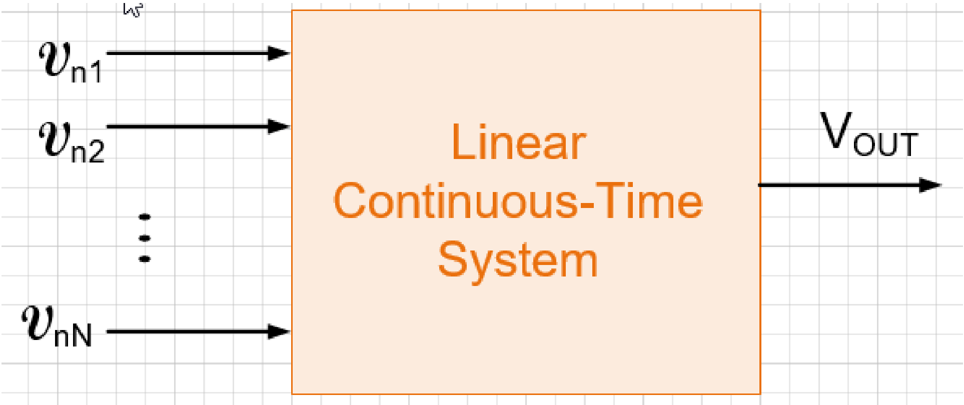
$$V_{OUT}(s) = \sum_{i=1}^{N} T_i(s) V_i(s)$$

If the noise sources are uncorrelated with spectral density S_1, … S_N, the spectral density and the RMS noise voltage at the output are given by the equations:

$$S_{OUT} = \sum_{i=1}^{N} S_i \bullet \left|T_i(j\omega)\right|^2$$

$$v_{OUT\_RMS} = \sqrt{\int_{f=0}^{\infty} S_{OUT} df} = \sqrt{\int_{f=0}^{\infty} \sum_{i=1}^{N} S_i \bullet \left|T_i(j\omega)\right|^2 df}$$

Analysis of Noise in Filter Circuits



$$v_{OUT\_RMS} = \sqrt{\int_{f=0}^{\infty} S_{OUT} df} = \sqrt{\int_{f=0}^{\infty} \sum_{i=1}^{N} S_i \bullet |T_i(j\omega)|^2 df}$$

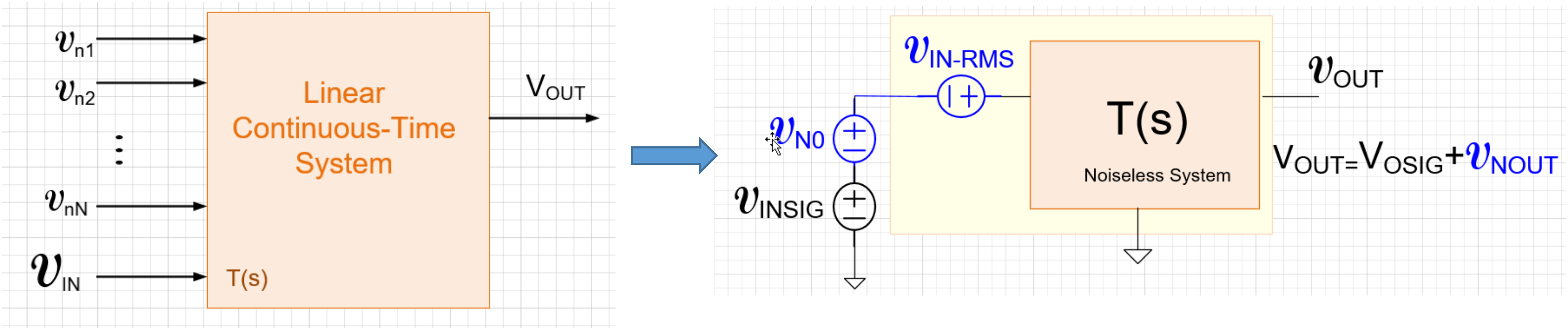
A noise analysis in the frequency domain can be easily run in Spectre to obtain the RMS noise voltage at the output

This can be referred back to the input by dividing by the gain from the input to the output to determine the input-referred SNR (see next page)

There is now a time-domain noise analysis capability in Cadence so actual time-domain noise analysis is possible

v_{N0} usually not part of the filter so affects system but not filter

Input-Referred Noise in Filter Circuits



$$v_{OUT\_RMS} = \sqrt{\int_{f=0}^{\infty} S_{OUT} df} = \sqrt{\int_{f=0}^{\infty} \sum_{i=1}^{N} S_i \bullet \left|T_i(j\omega)\right|^2 df}$$

Let T(s) be the transfer function from the input to the output. (usually T(s) will be distinct from each of the noise transfer functions).

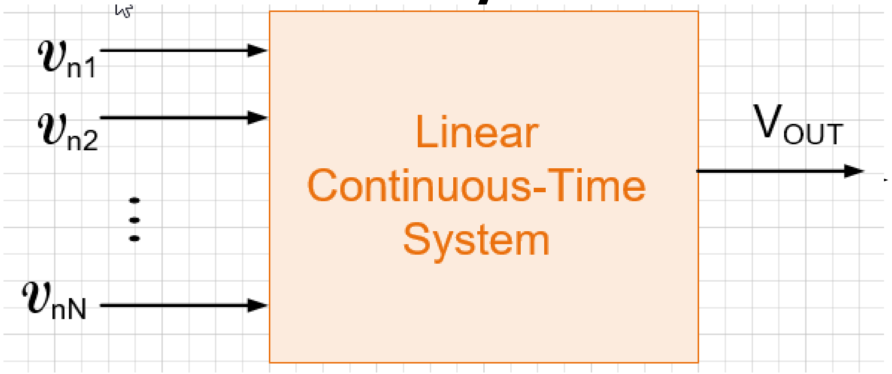
The input-referred noise spectral density is given by the expression

$$S_{IN} = \frac{S_{OUT}}{\left|T(j\omega)^2\right|}$$

The input-referred RMS voltage is thus given by

$$v_{IN\_RMS} = \sqrt{\int_{f=0}^{\infty} \frac{S_{OUT}}{\left|T(j\omega)\right|^2} df} = \sqrt{\int_{f=0}^{\infty} \sum_{i=1}^{N} S_i \bullet \frac{\left|T_i(j\omega)\right|^2}{\left|T(j\omega)\right|^2} df}$$

Relationship between frequency domain and time domain noise analysis



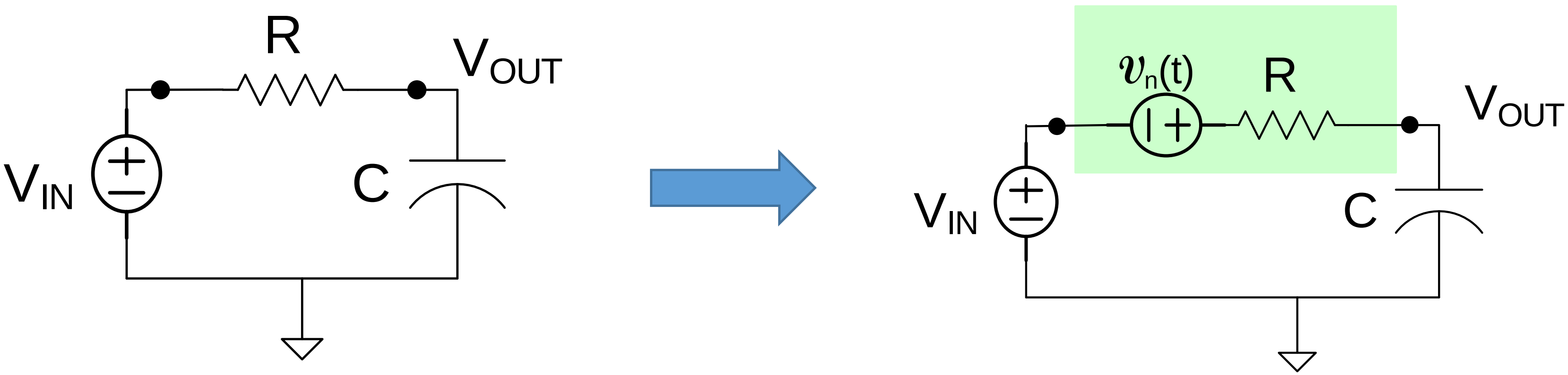
$$v_{OUT\_RMS} = \sqrt{\int_{f=0}^{\infty} S_{OUT} df} = \sqrt{\int_{f=0}^{\infty} \sum_{i=1}^{N} S_i \bullet \left|T_i(j\omega)\right|^2 df}$$

$$V_{RMS\_OUT} = E\left(\sqrt{\lim_{T\to\infty}\left(\frac{1}{T}\int_0^T V_{OUT}^2(t)dt\right)}\right) \simeq \sqrt{\lim_{T\to\infty}\left(\frac{1}{T}\int_0^T V_{OUT}^2(t)dt\right)}$$

Parseval's Theorem

$$V_{RMS\_OUT} = v_{OUT\_RMS}$$

Example: First-Order RC Network

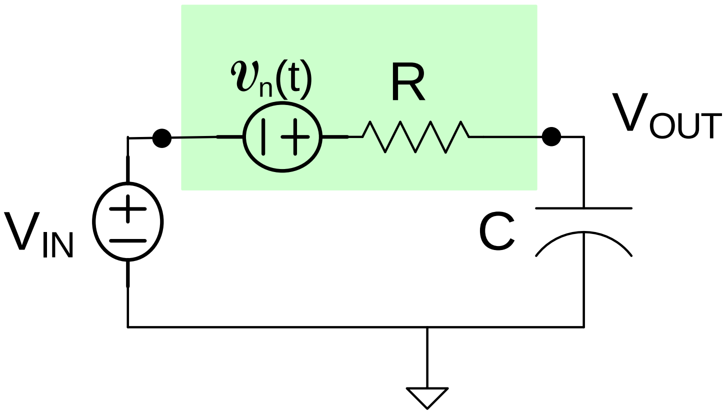


$$T(s) = \frac{1}{1+RCs}$$

$$S_{VOUT} = 4kTR\left(\frac{1}{1+(RC\omega)^2}\right)$$

$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}}\, df} = \sqrt{\int_{f=0}^{\infty} \frac{4kTR}{1+\omega^2R^2C^2}\, df}$$

Example: First-Order RC Network



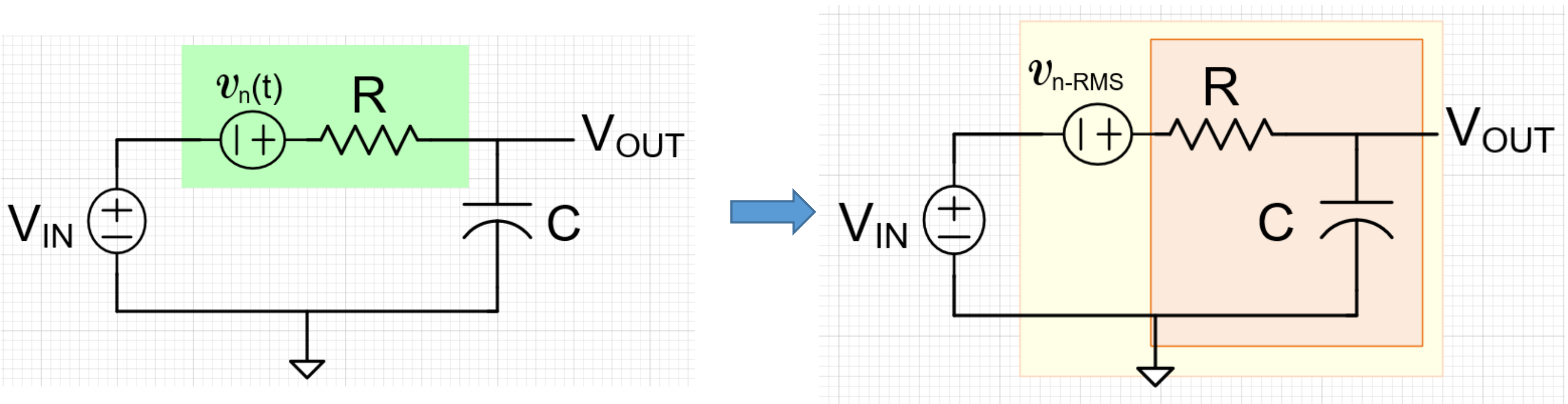
$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}}\,df} = \sqrt{\int_{f=0}^{\infty} \frac{4kTR}{1+\omega^2 R^2 C^2}\,df}$$

From a standard change of variable with a trig identity, it follows that

$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}}\,df} = \sqrt{\frac{kT}{C}}$$

- Note the continuous-time noise voltage has an RMS value that is independent of R
- The noise contributed by the resistor is dependent only upon the capacitor value C
- This is often referred to at kT/C noise and it can be decreased at a given T only by increasing C

Example: First-Order RC Network



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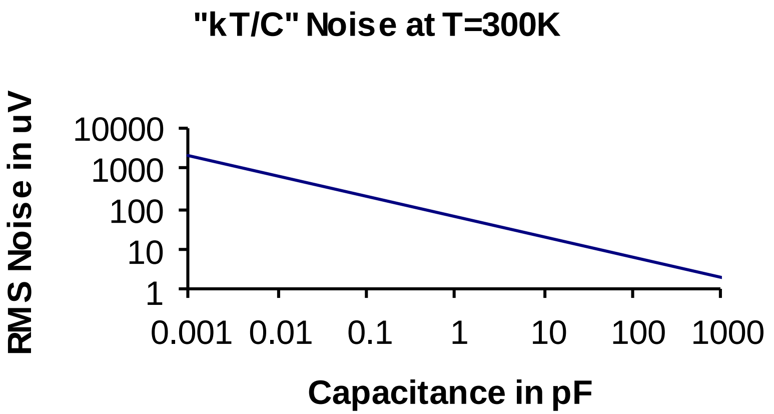
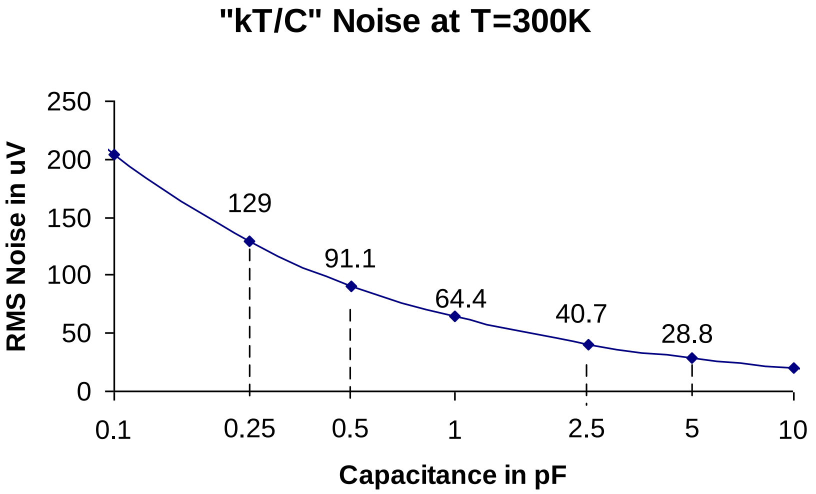
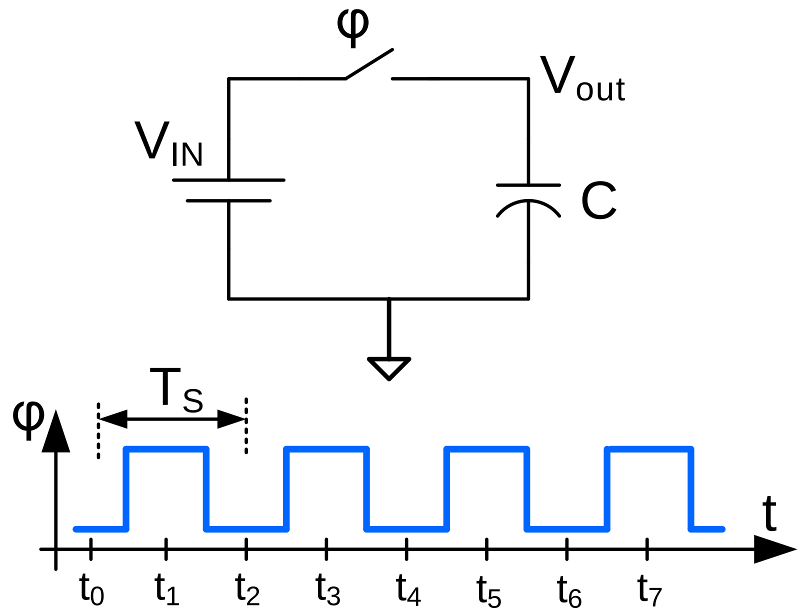
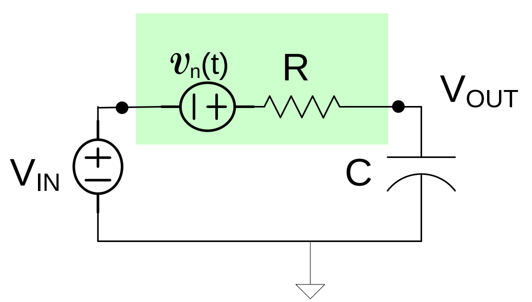
Noise Associated with Capacitors



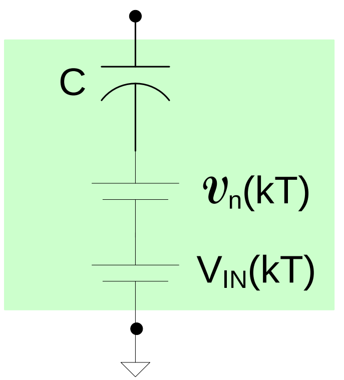


Example: Switched Capacitor Sampler



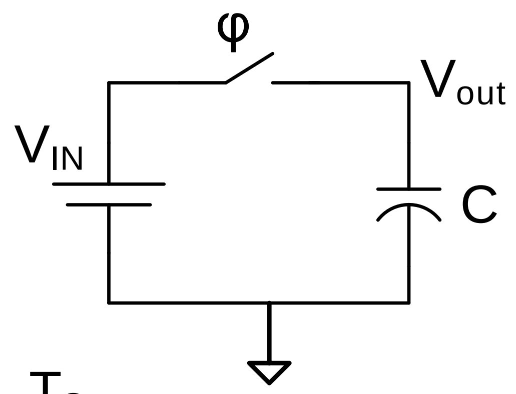


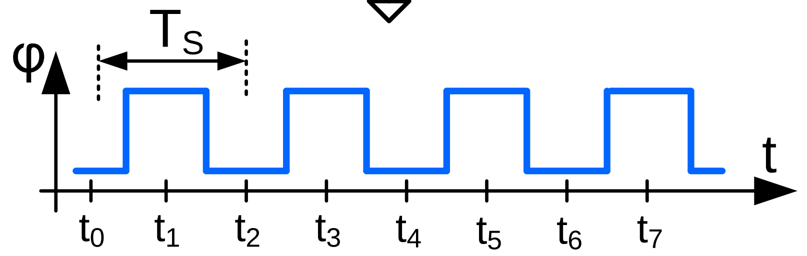
Track mode

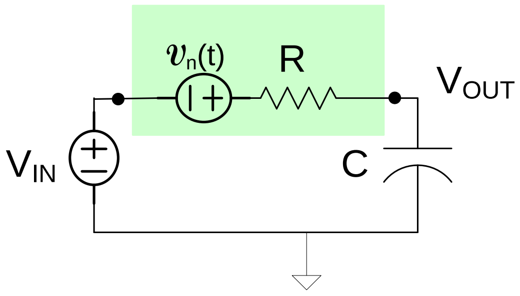


Hold mode

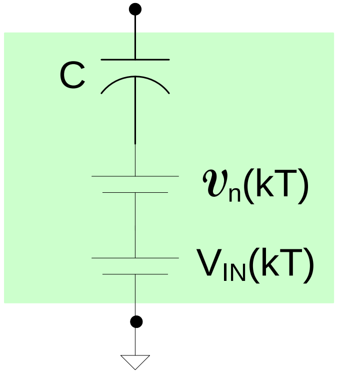
Example: Switched Capacitor Sampler





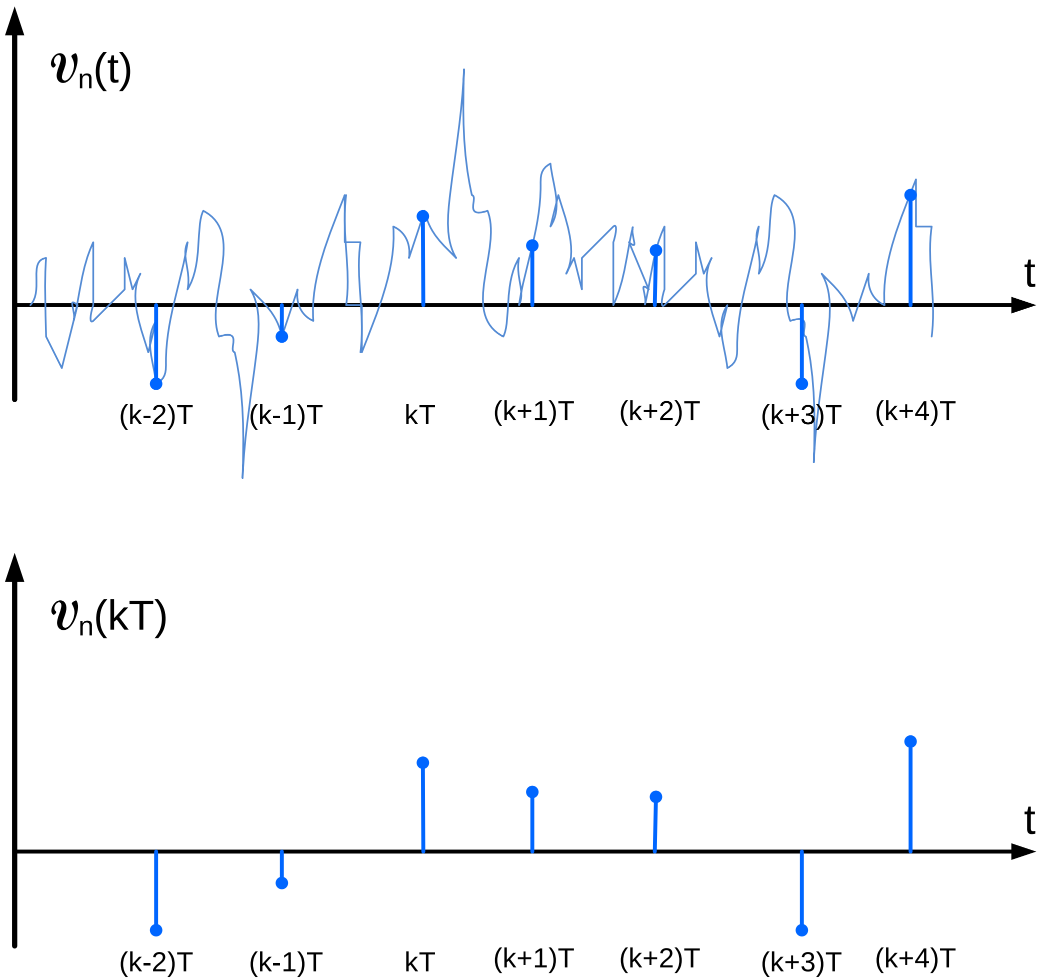


Track mode



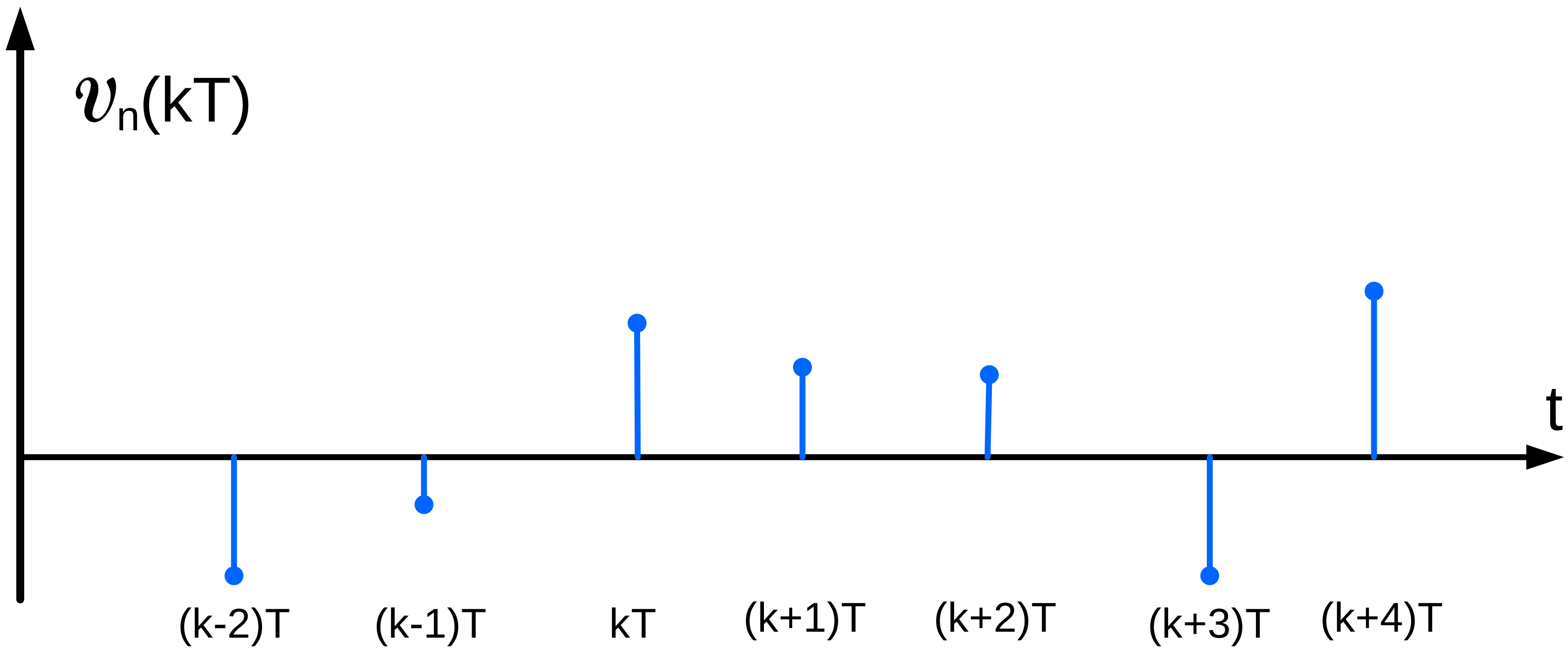
Hold mode

Example: Switched Capacitor Sampler



$v_n(kT)$ is a discrete-time sequence obtained by sampling a continuous-time noise waveform

Characterization of a noise sequence

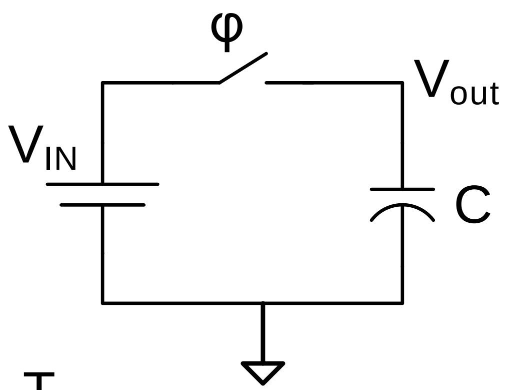


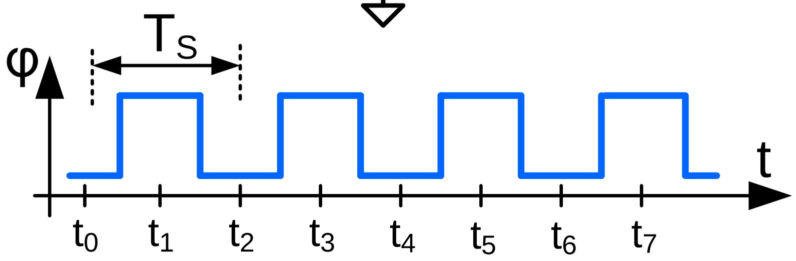
$$\hat{v}_{RMS} = E\left(\sqrt{\lim_{N\to\infty}\left(\frac{1}{N}\sum_{k=1}^{N} v^2(kT)\right)}\right) \underset{N\,large}{\simeq} \sqrt{\frac{1}{N}\sum_{k=1}^{N} v^2(kT)}$$

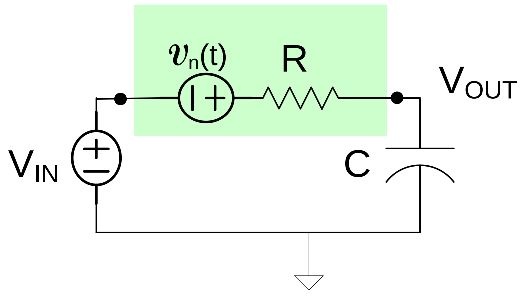
Theorem If $\mathcal{V}(t)$ is a continuous-time zero-mean noise source and <$\mathcal{V}(kT)$> is a sampled version of $\mathcal{V}(t)$ sampled at times T, 2T, …. then the RMS value of the continuous-time waveform is the same as that of the sampled version of the waveform. This can be expressed as $\mathcal{V}_{RMS} = \hat{\mathcal{V}}_{RMS}$

Theorem If $\mathcal{V}(t)$ is a continuous-time zero-mean noise signal and <$\mathcal{V}(kT)$> is a sampled version of $\mathcal{V}(t)$ sampled at times T, 2T, …. then the standard deviation of the random variable $\mathcal{V}(kT)$, denoted as $\sigma_{\hat{V}}$ satisfies the expression $\sigma_{\hat{V}} = \mathcal{V}_{RMS} = \hat{\mathcal{V}}_{RMS}$

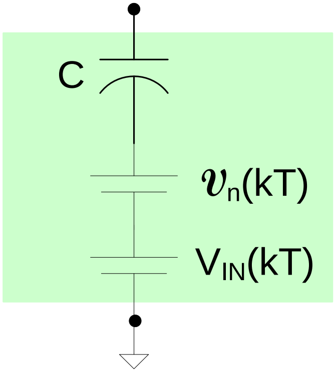
Example: Switched Capacitor Sampler







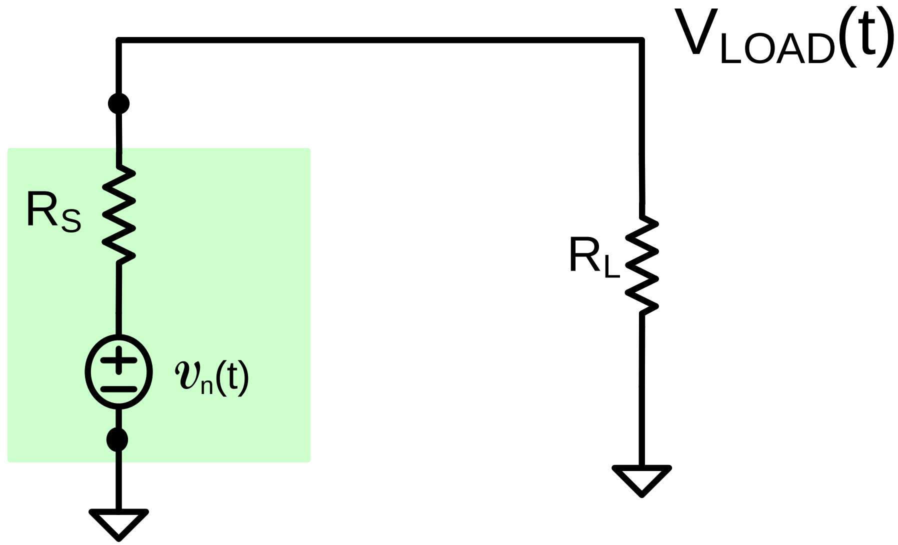
Track mode



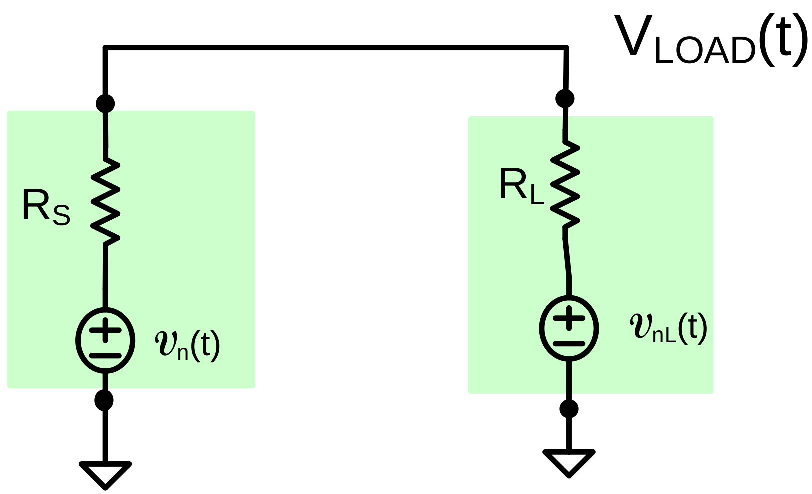
Hold mode

$$v_{n_{RMS}} = \sqrt{\frac{kT}{C}}$$

What is the RMS value of the output noise voltage due to the noise on R_S?



What is the RMS value of the output noise voltage due to the noise on R_L and R_S?



Stay Safe and Stay Healthy !

End of Lecture 36